

# A Formal Semantics of the Core DOM in Isabelle/HOL

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## ABSTRACT

At its core, the Document Object Model (DOM) defines a tree-like data structure for representing documents in general and HTML documents in particular. It is the heart of any modern web browser.

Formalizing the key concepts of the DOM is a prerequisite for the formal reasoning over client-side JavaScript programs and for the analysis of security concepts in modern web browsers.

We present a formalization of the core DOM, with focus on the *node-tree* and the operations defined on node-trees, in Isabelle/HOL. We use the formalization to verify the functional correctness of the most important functions defined in the DOM standard. Moreover, our formalization is (1) *extensible*, i.e., can be extended without the need of re-proving already proven properties and (2) *executable*, i.e., we can generate executable code from our specification.

## CCS CONCEPTS

• **Information systems** → **Markup languages**; Document structure; • **Software and its engineering** → **Software verification**; **Semantics**; *Formal software verification*;

## KEYWORDS

Document Object Model; DOM; Formal Semantics; Isabelle/HOL

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## 1 INTRODUCTION

In a world in which more and more applications are offered as services on the internet, web browsers start to take on a similarly central role in our daily IT infrastructure as operating systems. Thus, web browsers should be developed as rigidly and formally as operating systems. While formal methods are a well-established technique in the development of operating systems (see, e. g., Klein [15] for an overview), there are few proposals for improving the development of web browsers using formal approaches [2, 9, 12, 17].

As a first step towards a verified client-side web application stack, we model and formally verify the Document Object Model (DOM) in Isabelle/HOL. The DOM [21, 23] is *the* central data structure of all modern web browsers. At its core, the Document Object

Model (DOM) defines a tree-like data structure for representing documents in general and HTML documents in particular. Thus, the correctness of a DOM implementation is crucial for ensuring that a web browser displays web pages correctly. Moreover, the DOM is the core data structure underlying client-side JavaScript programs, i. e., client-side JavaScript programs are mostly programs that read, write, and update the DOM.

In more detail, we formalize the core DOM as a shallow embedding [14] in Isabelle/HOL. Our formalization is based on a typed data model for the *node-tree*, i. e., a data structure for representing XML-like documents in a tree structure. Furthermore, we formalize a typed heap for storing (partial) node-trees together with the necessary consistency constraints. Finally, we formalize the operations (as described in the DOM standard [23]) on this heap that allow manipulating node-trees.

Our machine-checked formalization of the DOM node tree [23] has the following desirable properties:

- (1) It provides a *consistency guarantee*. Since all definitions in our formal semantics are conservative and all rules are derived, the logical consistency of the DOM node-tree is reduced to the consistency of HOL.
- (2) It serves as a *technical basis for a proof system*. Based on the derived rules and specific setup of proof tactics over node-trees, our formalization provides a generic proof environment for the verification of programs manipulating node-trees.
- (3) It is *executable*, which allows to validate its compliance to the standard by evaluating the compliance test suite on the formal model and
- (4) It is *extensible* in the sense of [5], i. e., properties proven over the core DOM do not need to be re-proven for object-oriented extensions such as the HTML document model.

Finally, we show the correctness of the functions for manipulating the DOM w.r.t. the assumptions made in the standard.

After introducing Isabelle and higher-order logic in Sect. 2, we introduce the formal data model of the DOM and operations over the DOM in Sect. 3. In Sect. 4, we formalize the requirements for a valid heap, followed by the discussion of the verification of important properties of DOM operations in Sect. 5. Finally, we discuss related work (Sect. 6) and draw conclusions (Sect. 7).

## 2 FORMAL AND TECHNICAL BACKGROUND

In this section, we will outline the underlying logical and methodological framework of our formalization of the DOM node-tree.

### 2.1 Higher-Order Logic and Isabelle

Isabelle [16] is a generic theorem prover implemented in the functional programming language SML. Isabelle/HOL is the instance

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of Isabelle supporting Higher-order logic (HOL) [1, 8]. It supports conservativity checks of definitions, datatypes, primitive and well-founded recursion, and powerful generic proof engines based on rewriting and tableau provers.

HOL is a classical logic with equality enriched with total polymorphic higher-order functions. HOL is strongly typed, i.e., each expression  $e$  has a type  $'a$ , written  $e :: 'a$ . In Isabelle, we denote type variables with a prime (e.g.,  $'a$ ) instead of Greek letters (e.g.,  $\alpha$ ) that are usually used in textbooks. The type constructor for the function space is written using infix notation:  $'a \Rightarrow 'b$ . HOL is centered around the extensional logical equality  $_ = _$  with type  $'a \Rightarrow 'a \Rightarrow \text{bool}$ , where  $\text{bool}$  is the fundamental logical type.

Functions in HOL are curried and pure, i.e., they take exactly one argument, return exactly one result, and cannot produce side-effects. To simulate functions with more than one argument, we let these functions again return a function, until it will finally return a non-function. Therefore, when reading curried function definitions, it can be helpful to interpret the chain of function definitions in the following way: the last type definition represents the “return value” of the function, whereas the other types in the chain represent arguments to the function. When modeling stateful functions, such as in our case, we usually define functions that take an argument that represents the state and return an updated version (i.e., a map that contains an additional entry) that represents the state change.

The type discipline rules out paradoxes such as Russel’s paradox in untyped set theory. Sets of type  $'a$   $\text{Set}$  can be defined isomorphic to functions of type  $'a \Rightarrow \text{bool}$ ; the element-of-relation  $_ \in _$  has the type  $'a \Rightarrow 'a \text{ set} \Rightarrow \text{bool}$  and corresponds basically to the function application; the set comprehension  $\{ _ \ . _ \}$  (usually written  $\{ _ | _ \}$  in textbooks) has type  $'a \text{ set} \Rightarrow ('a \Rightarrow \text{bool}) \Rightarrow 'a \text{ set}$  and corresponds to the  $\lambda$ -abstraction.

Isabelle/HOL allows for defining abstract datatypes. For example, the following statement introduces the option type:

```
datatype 'a option = None | "Some 'a"
```

Besides the *constructors* `None` and `Some`, there is the match-operation `case x of None  $\Rightarrow$  F | Some a  $\Rightarrow$  G a`. The option type allows us to represent *partial functions* (often called *maps*) as total functions of type  $'a \Rightarrow 'b \text{ option}$ . For this type, we introduce the shorthand  $'a \rightarrow 'b$ . We define `dom f`, called the *domain* of a partial function  $f$ , by the set of all arguments of  $f$  that do not yield `None`.

We also make use of the sum type,  $'a + 'b$ , and the product type,  $'a \times 'b$ . With the sum type, it is possible to express tuples, which, for example, can be used to achieve a similar result to returning a tuple from a function. The product type represents either  $'a$  or  $'b$ , and is useful to model errors, as it allows to let functions return either some result on a successful calculation, or return an error.

When extending logics, two approaches can be distinguished: the *axiomatic method* on the one hand and *conservative extensions* on the other. Extending the HOL core via axioms, i.e., introducing new, *unproven* laws seems to be the easier approach but it usually leads easily to inconsistency; given the fact that in any major theorem proving system the core theories and libraries contain several thousand theorems and lemmas, the axiomatic approach is worthless in practice. In contrast, a conservative extension introduces new constants (via *constant definitions*) and types (*type definitions*)

only via a particular schema of axioms; the (meta-level) proof that axioms of this schema preserve consistency can be found in [10].

## 2.2 Shallow Embeddings vs. Deep Embeddings

We are now concerned with the question *how* a language is represented in a logic. Two techniques are distinguished: First, *deep embeddings* represent the abstract syntax as a datatype and define a semantic function  $I$  from syntax to semantics. Second, *shallow embeddings* define the semantics directly; each construct is represented by some function on a semantic domain.

Assume we want to embed a simple logical language `BOOL`, consisting of the two logical operators `_ and _` and `_ or _`, into HOL. The semantics  $I : \text{expr} \rightarrow \text{env} \rightarrow \text{bool}$  is a function that maps `BOOL` expressions and environments to *bool*, where *environments*  $\text{env} = \text{var} \rightarrow \text{bool}$  maps variables to *bool* values. Using a shallow embedding, we define directly:

$$x \text{ and } y \equiv \lambda e \bullet x e \wedge y e \quad x \text{ or } y \equiv \lambda e \bullet x e \vee y e$$

Shallow embeddings allow for direct definitions in terms of semantic domains and operations on them. In a deep embedding, we have to define the syntax of `BOOL` as a recursive datatype:

$$\text{expr} = \text{var } \text{var} \mid \text{expr and expr} \mid \text{expr or expr}$$

and the explicit semantic function  $I$ :

$$\begin{aligned} I[\text{var } x] &= \lambda e \bullet e(x) \\ I[x \text{ and } y] &= \lambda e \bullet I[x] e \wedge I[y] e \\ I[x \text{ or } y] &= \lambda e \bullet I[x] e \vee I[y] e \end{aligned}$$

This example reveals the main drawback of deep embeddings: the language is more distant to the underlying meta language HOL, i.e. semantic functions represent obstacles for deduction. However; for analyzing certain meta-theoretic analysis, deep-embeddings have advantages. Since we are interested in a concise semantic description of the DOM and efficient proof support (and we are not interested in meta-theoretic proofs), we chose a shallow embedding.

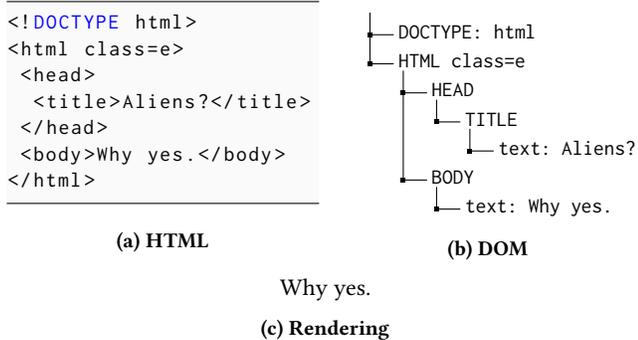
## 3 FORMALIZING THE DOM

In this section, we will present our formalization of the core DOM which follows the WHATWG specification [23], the updated version of the W3C DOM 4 standard [21]. This includes the definition of the tree-like data structure for representing documents and a set of functions for creating and modifying a document.

### 3.1 The Core DOM Data Model: The Node-Tree

The main purpose of the DOM is to provide the data structure for managing tree-structured documents, e.g., following the HTML or XML standard. Fig. 1 illustrates a small example: Fig. 1a shows the textual representation of a simple document (using HTML as syntax), Fig. 1b shows the visualization of the DOM *node-tree*, and Fig. 1c shows the rendered output (e.g., in a web browser).

As the DOM models a tree-like data structure, it is not a surprise that the core datatypes of the DOM specifications are `Document` and the datatype `Node` with the two specializations `Element` and `CharacterData`. In our data model, we omitted attributes that can be computed from others, e.g., the parent attribute which represent



**Fig. 1: A simple example of a DOM: (a) shows a textual representation using HTML syntax, (b) a visualization of the node-tree of the DOM, and (c) shows the result of rendering this DOM, e.g., by a rendering engine of a web browser.**

the inverse relation already represented by the `childNodes` and `documentElement` attributes.

While the core idea of formalizing object-oriented data models in an extensible way<sup>1</sup> follows the construction presented in [3, 5], we differ significantly in aspects such as the modeling of typed pointers (references) and late binding of method invocations. Due to space constraints, we will not discuss this in this paper.

First, we start by defining abstract datatypes for typed pointers for the common super-class `Object` and the classes `Node`, `Element`, `CharacterData`, and `Document`:

#### datatype

```
'object_ptr object_ptr = Ext 'object_ptr
'node_ptr node_ptr = Ext 'node_ptr
'element_ptr element_ptr = Ref ref | Ext 'element_ptr
'character_data_ptr character_data_ptr = Ref ref
| Ext 'character_data_ptr
'document_ptr document_ptr = Ref ref | Ext 'document_ptr
```

The pointers to the abstract classes `object_ptr` and `node_ptr` only support a constructor for extensions; regular classes also have a constructor for the reference of the object itself. We use these datatypes to introduce type synonyms representing the actual pointer types for our DOM model (see Fig. 2).

The type polynomials are constructed in such a way that the HOL types for pointers of sub-classes in the object-oriented model are instances of the HOL type of their super-class. This is the key construction allowing an extensible formalization. For details, we are referring the reader to [5].

In the rest of this paper, we will use an underscore to denote the tuple of type variables of the type constructors for pointer and object types. For example, we will write `_ node_ptrCore_DOM` instead of

```
('node_ptr, 'element_ptr,
 'character_data_ptr) node_ptrCore_DOM
```

and assume that type variables of the same name are instantiated with the same types.

<sup>1</sup>This object-oriented form of extensibility allows us, e.g., to later extend the DOM model to a formal model of standards based on the DOM standard, such as HTML, without the need to re-prove properties over the DOM.

Second, we define HOL types representing objects using the record-package provided in Isabelle. Overall, we use the same construction of type polynomials to represent inheritance in HOL. Due to space reasons, we omit the technical details of the type construction. We refer interested readers to the Isabelle formalization [4]. For each class, we define one record:

```
record Object =
  nothing :: unit
record Node = Object +
  nothing :: unit
record _ Element = Node +
  tag_name :: tag_type
  child_nodes :: "_ node_ptrCore_DOM list"
  attributes :: attributes_type
record CharacterData = Node +
  data :: DOMString
record _ Document = Object +
  doctype :: doctype
  document_element :: "_ element_ptrCore_DOM option"
  disconnected_nodes :: "_ node_ptrCore_DOM list"
```

Due to technical constraints of the record package, we need to introduce an attribute `nothing` for classes that do not define at least one attribute themselves. Given these definitions, we can, e.g., define a `CharacterData` object as follows:

#### definition

```
"CharacterDataExample = (|Object.nothing = (),
                        Node.nothing = (),
                        data = 'Why yes.' |)"
```

Essentially, this models an object-oriented data model of a tree-like data structure, called *node-tree* in the DOM standard, where (1) the root of the tree is an instance of `Document`, (2) instances of the class `Element` can be internal nodes or leaves, and (3) instances of the class `CharacterData` can only appear as leaves.

Finally, we define a heap for storing node-trees, i.e., instances of our DOM data model. A DOM heap is a map from object pointers to objects:

```
type_synonym _ heapCore_DOM
= "_ object_ptrCore_DOM  $\rightarrow$  _ ObjectCore_DOM"
```

Where `_ ObjectCore_DOM` is the type synonym for the instantiated super-type of `object` (similar to the construction for pointers).

Fig. 3 illustrates how the simple document from our example in Fig. 1 can be expressed in our formal DOM heap.

## 3.2 Operations and Queries on Node-Trees

In the following, we will define the core DOM methods for creating, querying, and modifying the node-trees that are stored in a DOM heap. We define the following functions formally in Isabelle/HOL. Fig. 4 provides an overview of their formal type signatures.

All operations are defined over the DOM heap, i.e., they take a heap as input and return either an exception or a tuple containing the return value and a new heap:

```
type_synonym (_, 'result) dom_prog = "_ dom_heapCore_DOM
 $\Rightarrow$  exception + ('result  $\times$  _ dom_heapCore_DOM)"
```

---

### type\_synonym

```
('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr, 'shadow_root_ptr) object_ptrCore_DOM
= "((('element_ptr element_ptr + 'character_data_ptr character_data_ptr + 'node_ptr) node_ptr
+ 'document_ptr document_ptr + 'shadow_root_ptr shadow_root_ptr + 'object_ptr) object_ptr"
('node_ptr, 'element_ptr, 'character_data_ptr) node_ptrCore_DOM
= "('element_ptr element_ptr + 'character_data_ptr character_data_ptr + 'node_ptr) node_ptr"
'element_ptr element_ptrCore_DOM = "'element_ptr element_ptr"
'character_data_ptr character_data_ptrCore_DOM = "'character_data_ptr character_data_ptr"
'document_ptr document_ptrCore_DOM = "'document_ptr document_ptr"
```

---

**Fig. 2: The HOL types of our extensible, typed pointers for the core DOM. The type polynomials ensure that pointers for sub-classes in the object-oriented data model are instances of the HOL type of the pointer of their super-class.**

---

### definition "test\_heap = map\_of [

```
(cast (document_ptr.Ref 1), cast (|Object.nothing = (), doctype = 'html',
document_element = Some (element_ptr.Ref 1), disconnected_nodes = [] |)),
(cast (element_ptr.Ref 1), cast (|Object.nothing = (), Node.nothing = (),
tag_name = 'html', child_nodes = [cast (element_ptr.Ref 2), cast (element_ptr.Ref 4)],
attributes = map_of [('class', 'e')], shadow_root_opt = None |)),
(cast (element_ptr.Ref 2), cast (|Object.nothing = (), Node.nothing = (),
tag_name = 'head', child_nodes = [cast (element_ptr.Ref 3)],
attributes = empty, shadow_root_opt = None |)),
(cast (element_ptr.Ref 3), cast (|Object.nothing = (), Node.nothing = (), tag_name = 'title',
child_nodes = [cast (character_data_ptr.Ref 1)], attributes = empty, shadow_root_opt = None |)),
(cast (character_data_ptr.Ref 1), cast (|Object.nothing = (), Node.nothing = (), data = 'Aliens?' |)),
(cast (element_ptr.Ref 4), cast (|Object.nothing = (), Node.nothing = (), tag_name = 'body',
child_nodes = [cast (character_data_ptr.Ref 2)], attributes = empty, shadow_root_opt = None |)),
(cast (character_data_ptr.Ref 2), cast (|Object.nothing = (), Node.nothing = (), data = 'Why yes.' |))
]"
```

---

**Fig. 3: The formal representation of a heap containing our simple example DOM (recall Fig. 1).**

All operations result in an exception if an argument is invalid, e. g., a pointer that does not represent a valid object in the current heap. We use a heap and error monad for modeling exceptions. This allows us to define composite methods similar to stateful programming in Haskell, but also to stay close to the official specification.

The function `create_element` takes an (owner)document and the tag name of the new element. It returns the updated heap that includes the new element with no children and no attributes along with a reference to the new element, which is stored in the first free location in the heap. This ensures that it will not change any existing locations in the heap, which we will prove later. Additionally, the new element is added to the list of disconnected nodes of the given document, as it is not yet part of the node tree.

The function `get_child_nodes` takes a heap and a pointer to a node and returns a list of pointers to its children. For elements, it returns the children list that is stored in the datatype. For text nodes, it returns the empty list. For documents, we convert their document element into the appropriate node list.

The function `get_attribute` looks up the given attribute in the element’s attribute map. It returns `Some attr` if there exists an attribute with the given key, and `None` otherwise. The official specification also has a concept called “reflected content attribute,” which basically returns the stored attribute of the same name, but returns the empty string if the attribute is not present.

The function `set_attribute` updates the given attribute of the pointer in the heap. In the official specification, it is not allowed to set the attribute to `None` or `null`, respectively, to delete the attribute. We generalize this definition by allowing this.

The function `get_parent_node` takes a pointer to a node and returns a pointer to its parent, or `None`, if the node does not have a parent. The case where a node does not have a parent can only occur in *disconnected* node-trees, which we will discuss later. Our API does not accept documents, since they can never have a parent. Having the types as narrow as possible will enable easier proofs. The function `get_parent_node` is an example of a method where the official specification leaves much room for interpretation regarding the implementation. It neither provides an algorithm explaining to how obtain a parent, given a node, nor does it specify that the parent reference should be stored in the objects. To avoid specifying additional consistency constraints that would be needed if both children and parent references were to be stored, we implemented `get_parent_node` by searching the whole heap for any node whose `get_child_nodes` contains the given reference.

The function `remove_child` is rather close to the official specification; if `child’s parent` is different from the passed parent, then we “throw” a `NotFoundError`. Otherwise, we add the removed child to the disconnected node list of its owner document and remove it from either the `document_element` or the `child_nodes` attribute.

```

create_element  :: "_ tag_type => _ document_ptrCore_DOM => _ dom_prog"
get_attribute   :: "_ element_ptrCore_DOM => attributes_key => _ dom_prog"
set_attribute   :: "_ element_ptrCore_DOM => attributes_key => attributes_value option => _ dom_prog"
get_child_nodes :: "_ object_ptrCore_DOM => _ dom_prog"
get_parent      :: "_ node_ptrCore_DOM => _ dom_prog"
remove_child    :: "_ object_ptrCore_DOM => _ node_ptrCore_DOM => _ dom_prog"
get_element_by_id :: "_ object_ptrCore_DOM => attributes_value => _ dom_prog"
adopt_node      :: "_ document_ptrCore_DOM => _ node_ptrCore_DOM => _ dom_prog"
insert_before   :: "_ object_ptrCore_DOM => _ node_ptrCore_DOM => _ node_ptrCore_DOM option => _ dom_prog"

```

Fig. 4: The formal type signatures of the methods for creating, querying, and modifying the core DOM.

The function `get_element_by_id` searches in tree order (depth-first, left-to-right) for the first element with the given id. Our definition is more general than the official specification, as we dropped the requirement that `get_element_by_id` should only be available on documents, which is a legacy requirement.

**3.2.1 Adopting Nodes.** The method `adopt_node` removes a node from its previous parent, if it had any, and assigns it to the new owner Document. First, it tries to retrieve the parent of the node to be adopted. If the node has a parent node, it removes the node from the children list, otherwise it removes it from the list of disconnected nodes of the previous owner document. Finally, the node is now added to the disconnected nodes of the new document.

```

definition adopt_node ::
  "_ document_ptrCore_DOM => _ node_ptrCore_DOM
  => _ dom_prog"
where
  "adopt_node document node = do {
    parent_opt ← get_parent node;
    (case parent_opt of
      Some parent => remove_child parent node
    | None => do {
      old_document ← get_owner_document (cast node);
      remove_from_disconnected_nodes old_document
        node});
    add_to_disconnected_nodes document node
  }"

```

**3.2.2 Inserting Nodes.** Using `insert_before`, one can insert arbitrary nodes (i.e., not necessarily in the same node-tree) from the heap into a node-tree:

```

definition insert_before ::
  "_ object_ptrCore_DOM => _ node_ptrCore_DOM
  => _ node_ptrCore_DOM option => _ dom_prog"
where
  "insert_before ptr node child = do {
    ensure_preinsertion_validity node ptr child;
    reference_child ← (if Some node = child
      then next_sibling node
      else return child);
    owner_document ← get_owner_document ptr;
    adopt_node owner_document node;
    insert_node ptr node reference_child
  }"

```

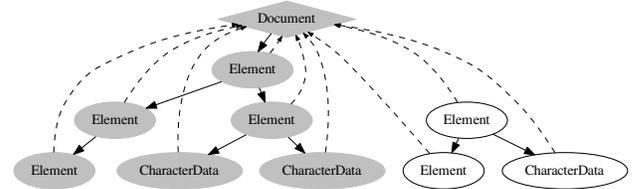


Fig. 5: DOM with a visible document (gray) and a runtime tree (white)

A node that should be inserted needs to fulfill certain well-formedness criteria. This is checked using the `ensure_preinsertion_validity` function which formalizes the concept of *pre-insertion validity* from the DOM standard. Then, the reference child needs to be determined, which is that node before which the to-be-inserted node should be placed. Then, we adopt the node into the (possibly new) node-tree and actually insert the node into either the `child_nodes` or `document_element` attributes.

## 4 WELL-FORMEDNESS OF THE DOM HEAP

Our DOM heap is a map from object pointer to object. While a map alone would allow numerous “illegal” heaps, two features of our formalization already rule out many misconfigurations: Our data model is typed and, thus, rules out illegal heaps such as one that contains a document that contains a character data object as its only child. Additionally, our data model omits some fields of the standard, such as `parentNode`, which we calculate by using the heap and `get_child_nodes`.

Still, some possible illegal heap configurations remain, such as one with a cyclic `get_child_nodes` relationship. Thus, we need further well-formedness constraints and we need to show that the DOM methods preserve the well-formedness. We will now introduce predicates that validate whether a given heap conforms to the standard.

### 4.1 The Owner Document

The DOM specifications requires that each node is owned by exactly one document, its owner document. Moreover, each node participates in a tree w.r.t. the `get_child_nodes`-relation. A DOM might—and usually will—consist of several trees, i.e., a DOM is a forest of trees. We call the tree that has the main document as root the *visible document*, as this is the part of the DOM that would be rendered, e.g., by a web browser.

Fig. 5 illustrates this relationship for an example: the gray nodes (connected by solid arrows that visualize the `get_child_nodes` relation) represent the visible document. The white nodes (connected by dotted arrows that visualize the `get_owner_document` relationship) are forming a temporary *runtime tree*. Runtime trees are not serialized (e. g., in an HTML or XML document) and only exist at runtime.

We define `get_owner_document` of a node to be the root, if the root is an document; otherwise, we return that document whose `disconnected_nodes` contains said node. In order for this definition to be well-formed, we need the following predicate:

```
definition owner_document_valid :: "_ heapCore_DOM ⇒ bool"
where "owner_document_valid h = (
  {node_ptr. ∀doc_ptr disc_node_ptrs.
    (h ⊢ get_disconnected_nodes doc_ptr →r disc_node_ptrs)
    → node_ptr ∈ set disc_node_ptrs}
= {node_ptr. ∀ptr.
  (h ⊢ get_root_node (cast node_ptr) →r ptr)
  → ¬is_document_ptr_kind ptr})"
```

This predicate guarantees us that the set of nodes in all `disconnected_nodes` fields is exactly the set of nodes that do not have a document as their root.

## 4.2 Restricting DOMs to Trees

So far, we do not restrict the relation given by `get_child_nodes` to be *acyclic*, which is possible since we use pointers. To prevent this, we can use the following predicate:

```
definition acyclic_heap :: "_ heapCore_DOM ⇒ bool"
where "acyclic_heap h = acyclic {(parent, child). ∀children.
  (h ⊢ get_child_nodes parent →r children)
  → child ∈ cast ' set children}"
```

We leverage the definition of acyclicity on relations, i.e., a set of tuples. Our relation contains all pointers `parent` and `child` where `child` is in the set of children of `parent`.

## 4.3 Node Sharing

The DOM standard assumes that a node cannot be the child of more than one node. This property of heaps is informally implied by the official standard, and all tree-modifying methods ensure that such a DOM cannot be built. We, however, must deal with all heaps that conform to our heap type. Therefore, in addition to our heap predicate that guarantees us that all trees in our heap are acyclic, we need a predicate that prevents the nodes from having more than one parent. Therefore, we formally define another heap predicate:

```
definition maximal_one_parent :: "_ heapCore_DOM ⇒ bool"
where "maximal_one_parent h = (∀node_ptr.
  (length (sorted_list_of_set {parent. ∀children.
    (h ⊢ get_child_nodes parent →r children)
    → node_ptr ∈ set children})) ≤ 1)"
```

The definition checks whether for any node, the set of possible parents (i.e., pointers whose children contains said node), contains exactly zero or one parents.

## 4.4 Pointer Validity

Moreover, we need to ensure that objects do not contain invalid pointers (e. g., pointers that do not point to an object stored in the heap of the same type). Otherwise, whenever we work with our pointers, we would have to deal with the possibility of a “null-pointer exception”. Thus, we require:

```
definition all_ptrs_in_heap :: "_ heapCore_DOM ⇒ bool"
where "all_ptrs_in_heap h = ((∀ptr children.
  (h ⊢ get_child_nodes ptr →r children)
  → set children ⊆ node_ptr_kinds h)
  ∧ (∀doc_ptr disc_node_ptrs.
  (h ⊢ get_disconnected_nodes doc_ptr →r disc_node_ptrs)
  → set disc_node_ptrs ⊆ node_ptr_kinds h))"
```

The only place where we can find pointers (without arbitrarily constructing them, which should be avoided) is in one of the datatype fields. Therefore, for all pointers in the heap, we retrieve the corresponding object, and check whether all pointers stored in applicable fields (`childNodes`, `document_element`, and `disconnected_nodes`) are present in the heap.

## 4.5 Heaps are Strongly Typed

As we model typed pointers and objects, we want to assure that a pointer of a certain type actually maps to an object of the related type in a given heap, e. g., that a `document_ptr` actually maps to a document. The following predicate assures us that this holds for the whole heap:

```
definition matches_heapCore_DOM :: "_ heapCore_DOM ⇒ bool"
where "matches_heapCore_DOM =
  (∀doc_ptr ∈ document_ptr_kinds heap.
  the (get (cast doc_ptr) is_document_kind heap))
  ∧ ..."
```

Similarly to document, the definition also contains checks for the other classes, which we omitted here due to space constraints. The definition checks whether for all, e.g., document pointers, the heap actually returns an object for which `is_document_kind` holds.

## 4.6 No Multi-Edges

The `childNodes` and `disconnected_nodes` attributes are of type list. Thus, they may contain duplicates, i.e., the same pointer multiple times. This can lead to strange effects, such as that after `remove_child` has been called, the pointer still is in the list. This behavior is not addressed by the official specification. We make this requirement explicit:

```
definition distinct_lists :: "_ heapCore_DOM ⇒ bool"
where "distinct_lists h = ((∀ptr children.
  (h ⊢ get_child_nodes ptr →r children)
  → distinct children)
  ∧ (∀doc_ptr disc_node_ptrs.
  (h ⊢ get_disconnected_nodes doc_ptr →r disc_node_ptrs)
  → distinct disc_node_ptrs))"
```

We retrieve the lists for every pointer in the heap and require that they are distinct. In Sect. 5.2, we will show a formal proof of the fact that `insert_node` actually can never lead to a `childNodes` list with duplicates.

**Table 1: Well-formedness properties of core heap methods**

	side-effect free	modifies only	preserves well-formedness
get_child_nodes	✓		
get_parent_node	✓		
get_element_by_id	✓		
get_attribute	✓		
create_element		✓	✓
set_attribute		✓	✓
remove_child		✓	✓
adopt_node		✓	✓
insert_before		✓	✓

## 4.7 Well-Formed Heaps

To put it all together, we define a well-formed heap as a heap that satisfies all discussed constraints:

```

definition heap_is_wellformed :: "_ heapCore_DOM ⇒ bool"
where "heap_is_wellformed h ⇔
  finite (object_ptr_kinds h) ∧ matches_heapCore_DOM h ∧
  owner_document_valid h ∧ acyclic_heap h ∧
  all_ptrs_in_heap h ∧ maximal_one_parent h ∧
  distinct_lists h"

```

## 5 REASONING OVER THE DOM

So far we only defined the DOM data structure, a heap for storing DOM instances, and methods over them. We now discuss the verification of these methods in the sense of formally proving that they preserve the well-formedness of the heap.

### 5.1 Properties of DOM Methods

*5.1.1 Well-formedness of the Heap Methods.* The DOM methods (see Sect. 3.2 and Table 1) can be divided into two categories: All query functions (starting with the prefix `get_`) use the heap to compute a value, but do not modify the heap. It is therefore easy to show that they preserve the well-formedness of the heap. For all other function, we have to formally prove their correctness w.r.t. preserving the well-formedness of the heap. If all methods preserve the well-formedness then we have shown that any exception-free sequence of DOM methods creates a well-formed DOM heap.

For all methods, we need to prove a lemma of this form:

```

lemma insert_before_preserves_wellformedness:
assumes "heap_is_wellformed h"
and "h ⊢ insert_before ptr new_child ref_child →h h'"
shows "heap_is_wellformed h'"

```

All variables in lemmas are all-quantified, meaning they can take all possible values of the corresponding type, only restricted by the statements in the assumption. As the predicate `heap_is_wellformed` is a conjunction of more specific predicates (e. g., `acyclic_heap`), we can split the proof for these lemmas into separate proofs that the methods are preserving those more specific conditions. We will discuss such a proof in more detail at the end of this section. Due to space limitations we will omit most proofs, for which we refer the reader to our full formalization.

*5.1.2 Heap Modifications are Local.* We want to ensure that heap-modifying functions do not modify the heap *arbitrarily*. Thus, we first introduce two predicates that characterize a function by specifying which locations (pointers) and fields are being read or written, respectively:

```

definition reads :: "(_ object_ptrCore_DOM
  × (_ object_ptrCore_DOM ⇒ _ heapCore_DOM
    ⇒ _ heapCore_DOM ⇒ bool)) set
  ⇒ _ dom_prog ⇒ bool"
where "reads S f ⇔ (∀h h' x. (h ⊢ f →r x)
  → (∀(ptr, P) ∈ S. P ptr h h') → (h' ⊢ f →r x))"

```

```

definition writes :: "(_ object_ptrCore_DOM
  × (_ object_ptrCore_DOM ⇒ _ heapCore_DOM
    ⇒ _ heapCore_DOM ⇒ bool)) set
  ⇒ _ dom_prog ⇒ bool"
where "writes S f ⇔ (∀h h'. (h ⊢ f →h h')
  → (∀ptr. ∀get ∈ Sg. (ptr, get) ∉ S → get ptr h h'))"

```

Both predicates take a set of pointers and predicates that assert something about the specified heap location in both the old and new heap. This will be, for example, a predicate checking whether the `attributes` field of `Element` will have the same value in both heaps. The `writes` predicate is characterized by these getter predicates that remain unaffected, which is why it references a set `Sg`, which contains all such getter predicates for the getters of our classes. For example, for the `get_attribute` and `set_attribute` DOM methods we prove the following:

```

lemma get_attribute_reads:
  "reads {(cast element_ptr,
    element_getter_preserved attributes)}
    (get_attribute element_ptr k)"

```

```

lemma set_attribute_writes:
  "writes {(cast element_ptr,
    element_getter_preserved attributes)}
    (set_attribute element_ptr k v)"

```

*5.1.3 Exceptions.* All our functions can throw exceptions, i. e., they return a sum type of exception and their real return type, which is a common way to model exceptions in functional languages. Therefore, we can provide lemmas that show under which preconditions our functions will return their normal result and not throw an exception. Most functions will throw an exception under exactly one circumstance: if they try to resolve a pointer on the given heap, but the heap does not have an object of the same type stored in that location, i. e., the lookup returns `None`. This is not surprising, since most functions will need to do something with the object, and not just the pointer to the object. For example, we show:

```

lemma set_attribute_ok:
assumes "matches_heapCore_DOM h"
and "ptr ∈ element_ptr_kinds h"
shows "h ⊢ ok (set_attribute ptr k v)"

```

---

```

lemma adopt_node_removes_child:
  assumes wellformed: "heap_is_wellformed h"
  and parent_known: "\parent.
    h \ get_parent node_ptr \r Some parent \implies
    is_known_ptrCore_DOM parent"
  and adopt_node: "h \ adopt_node owner_document node_ptr \to_h h2"
  and known_ptr: "is_known_ptrCore_DOM ptr"
  and children: "h2 \ get_child_nodes ptr \to_r children"
  shows "node_ptr \notin set children"

```

---

**Fig. 6: The method `adopt_node` removes the node that is to be adopted (proof in formalization document).**

---

```

lemma adopt_node_children_remain_distinct:
  assumes wellformed: "heap_is_wellformed h"
  and parent_known: "\parent.
    h \ get_parent node_ptr \to_r Some parent
    \implies is_known_ptrCore_DOM parent"
  and adopt_node: "h \ adopt_node owner_document node_ptr \to_h h2"
  and ptr_known: "is_known_ptrCore_DOM ptr"
  and children: "h2 \ get_child_nodes ptr \to_r children"
  shows "distinct children"

```

---

**Fig. 7: After using `adopt_node`, all children lists remain distinct (proof in formalization document).**

## 5.2 Proving Properties Over DOM Methods

Our DOM model allows us to prove properties of our specified DOM methods over arbitrary heaps. In proofs, the general line of arguing will usually utilize the fact that heap-modifying methods consist of a series of heap updates in single locations, whose proven properties we can utilize. We will show one example of such a proof to demonstrate how one can work with the formalization.

For example, we can prove that using `insert_before` does certainly never lead to duplicates in the node’s children list, even if a pointer is being inserted that is already in this node’s children. We express this property in our formalization as follows:

```

lemma insert_before_children_remain_distinct:
  assumes wellformed: "heap_is_wellformed h"
  and parent_known: "\parent.
    h \ get_parent new_child \to_r Some parent
    \implies is_known_ptrCore_DOM parent"
  and known: "is_known_ptrCore_DOM ptr"
  and insert_before:
    "h \ insert_before ptr new_child child_opt \to_h h2"
  shows "\ptr children. is_known_ptrCore_DOM ptr
    \implies h2 \ get_child_nodes ptr \to_r children
    \implies distinct children"

```

We first assume that we start with a wellformed heap. The next two assumptions guarantee us that `ptr` and the parent, if any, of `new_child` are of a known type — so either a `element_ptr`, `character_data_ptr`, or `document_ptr`. These two assumptions are necessary to prove something about one of our late-bound functions, `get_child_nodes` and `set_child_nodes`, later. The last assumption introduces `h2` as the heap after an application of `insert_before`.

The conclusion is to be read as follows: After the use of `insert_before` (`h2`), all lists of children of all known pointers will be distinct. Fig. 9 shows a formal proof sketch (i.e., a simplified excerpt of a formal proof using Isabelle’s proof language Isar [22]).

---

```

lemma insert_node_children_remain_distinct:
  assumes ptr_known: "is_known_ptrCore_DOM ptr"
  and insert_node: "h \ insert_node ptr new_child reference_child_opt
    \to_h h2"
  and "\children. h \ get_child_nodes ptr \to_r children
    \implies new_child \notin set children"
  and "\ptr children. is_known_ptrCore_DOM ptr
    \implies h \ get_child_nodes ptr \to_r children \implies distinct children"
  shows "\ptr children. is_known_ptrCore_DOM ptr
    \implies h2 \ get_child_nodes ptr \to_r children \implies distinct children"

```

---

**Fig. 8: After using `insert_node`, all children lists remain distinct if the child is not already in the children list into which it will be inserted (proof in formalization document).**

---

```

lemma insert_before_children_remain_distinct:
  assumes wellformed: "heap_is_wellformed h"
  and parent_known: "\parent.
    h \ get_parent new_child \to_r Some parent
    \implies is_known_ptrCore_DOM parent"
  and known: "is_known_ptrCore_DOM ptr"
  and insert_before:
    "h \ insert_before ptr new_child child_opt \to_h h2"
  shows "\ptr children. is_known_ptrCore_DOM ptr
    \implies h2 \ get_child_nodes ptr \to_r children
    \implies distinct children"
proof -
  obtain reference_child owner_document h' where
    reference_child: "h \ (if Some new_child = child_opt
      then next_sibling new_child
      else return child_opt) \to_r reference_child" and
    owner_doc: "h \ get_owner_document ptr \to_r owner_document" and
    h': "h' \ adopt_node owner_document new_child \to_h h'" and
    h2: "h' \ insert_node ptr new_child reference_child \to_h h2"
  by (insert assms, unfold insert_before_def) unfold_progs

  have "\ptr children. is_known_ptrCore_DOM ptr
    \implies h' \ get_child_nodes ptr \to_r children
    \implies distinct children"
  using adopt_node_children_remain_distinct parent_known
  using wellformed h' by blast
  moreover have "\ptr children. is_known_ptrCore_DOM ptr
    \implies h' \ get_child_nodes ptr \to_r children
    \implies new_child \notin set children"
  using h' parent_known wellformed adopt_node_removes_child
  by blast
  ultimately show "\ptr children. is_known_ptrCore_DOM ptr
    \implies h2 \ get_child_nodes ptr \to_r children
    \implies distinct children"
  by (metis insert_node_children_remain_distinct known h2)
qed

```

---

**Fig. 9: A formal proof that `insert_before` preserves the distinctness of the child nodes list**

*5.2.1 Proof Structure.* Recall the definition of `insert_before` in Sect. 3.2, which consists of two heap-modifying functions: `adopt_node`, which removes the new child from its old parent and updates the owner document, and `insert_node`, which appends the new child to the actual children list. The proof is therefore structured as follows:

First, we unroll the definition of `insert_before` to get a handle to the individual statements. Additionally, we obtain `h'`, which is the intermediate heap in between `adopt_node` and `insert_node`.

Second, we prove that after `adopt_node`, in addition to all children lists still being distinct (we only remove one child from one list), the child will not be part of any of these (as it has been removed from the only children list that contained

it). For this proof, we can use two properties of `adopt_node` that we proved earlier, `adopt_node_removes_child` (Fig. 6) and `adopt_node_children_remain_distinct` (Fig. 7).

Third, since we know that before the use of `insert_node` all children lists are distinct and do not contain the node that is to be inserted, we can prove that all children list will remain distinct, as we only insert the given node and nothing else. Again, for this proof, we can use a property that we have already proven about `insert_node`, `insert_node_children_remain_distinct` (Fig. 8).

## 6 RELATED WORK

To our knowledge, there are only very few formalizations of data structures for manipulating XML-like document structures available. Sternagel and Thiemann [19] present an “XML library” for Isabelle/HOL. The purpose of this library is to provide XML parsing and pretty printing facilities for Isabelle. As such, it is not a formalization of XML or XML-like data structures in Isabelle/HOL.

Our DOM typed formalization shares several design decision with the type-safe DOM API of Thiemann [20]. The most closely related works are [9, 17, 18] in which the authors present a non-executable, non-extensible, and non-mechanized operational semantics of a minimal DOM and show how this semantics can be used for Hoare-style reasoning for analyzing heaps of DOMs. The authors focus on providing a formal foundation for reasoning over client-side JavaScript programs that modify the DOM.

A more informal model of the DOM that focuses on the needs of building a static analysis tool for client-side JavaScript programs is presented by Jensen et al. [13]. This model does not focus on the DOM as such, instead the authors focus on the representation of HTML documents on top of the DOM.

Finally, there are several works, e. g., [2, 11, 12] on formalizing parts of web browsers for analyzing their security. These works use high-level specifications of web browsers and do not contain a formalization of the DOM itself.

## 7 CONCLUSION AND FUTURE WORK

We presented a typed formalization of the Document Object Model (DOM) in Isabelle/HOL. Technically, our formalization is an executable shallow embedding of the official specification of the WHATWG [23] and the W3C.

We see several lines of future work. We consider tightening the link between the formal specification and the actual implementations used by various web browsers as the most important line of future work. One promising approach to achieve this goal is the systematic generation of test cases from the formal specification using test case generation techniques that are integrated into Isabelle/HOL [6, 7]. The generated test cases can, as the already existing manually developed test cases, be used for validating the compliance of actual browser implementation.

Furthermore, there are two promising areas w.r.t. extending the scope of our formalization: first, formalizing and analyzing the “DOM with Shadow Roots,” i. e., the new component model proposed as part of the DOM standard of the WHATWG [23]. Second, using the extensibility of our formalization to add support for `HTML<Element>` (and its sub-types such as `HTMLIFrameElement`). As the concept of `iframes` is fundamental for restricting information

flow between parts of a website originating from different security domains, such a formalization would allow us to reason over web security properties in Isabelle/HOL.

**Availability.** The formalization is available under a 2-clause BSD license in the Archive of Formal Proofs [4]. A copy is also available at [https://git.logicalhacking.com/afp-mirror/Core\\_DOM](https://git.logicalhacking.com/afp-mirror/Core_DOM).

## REFERENCES

- [1] Peter B. Andrews. 2002. *Introduction to Mathematical Logic and Type Theory: To Truth through Proof*. Kluwer Academic Publishers.
- [2] Aaron Bohannon and Benjamin C. Pierce. 2010. Featherweight Firefox: Formalizing the Core of a Web Browser. In *Usenix Conference on Web Application Development (WebApps)*.
- [3] Achim D. Brucker. 2007. *An Interactive Proof Environment for Object-oriented Specifications*. Ph.D. Dissertation. ETH Zurich. ETH Dissertation No. 17097.
- [4] Achim D. Brucker and Michael Herzberg. 2018. *Archive of Formal Proofs* (2018). [http://www.isa-afp.org/entries/Core\\_DOM.shtml](http://www.isa-afp.org/entries/Core_DOM.shtml), Formal proof development. Submitted.
- [5] Achim D. Brucker and Burkhart Wolff. 2008. An Extensible Encoding of Object-oriented Data Models in HOL. *Journal of Automated Reasoning* 41 (2008), 219–249. Issue 3. <https://doi.org/10.1007/s10817-008-9108-3>
- [6] Achim D. Brucker and Burkhart Wolff. 2009. HOL-TestGen: An Interactive Test-case Generation Framework. In *Fundamental Approaches to Software Engineering*, Marsha Chechik and Martin Wirsing (Eds.). Number 5503 in LNCS. Springer-Verlag, 417–420. [https://doi.org/10.1007/978-3-642-00593-0\\_28](https://doi.org/10.1007/978-3-642-00593-0_28)
- [7] Achim D. Brucker and Burkhart Wolff. 2013. On Theorem Prover-based Testing. *Formal Aspects of Computing (FAC)* 25, 5 (2013), 683–721. <https://doi.org/10.1007/s00165-012-0222-y>
- [8] Alonzo Church. 1940. A formulation of the simple theory of types. *Journal of Symbolic Logic* 5, 2 (June 1940), 56–68.
- [9] Philippa Gardner, Gareth Smith, Mark J. Wheelhouse, and Uri Zarfaty. 2008. DOM: Towards a Formal Specification. In *Programming Language Technologies for XML*.
- [10] Mike J. C. Gordon and Tom F. Melham. 1993. *Introduction to HOL: a theorem proving environment for higher order logic*. Cambridge University Press.
- [11] Arjun Guha, Matthew Fredrikson, Benjamin Livshits, and Nikhil Swam. 2011. Verified Security for Browser Extensions. In *IEEE Symposium on Security and Privacy*. 115–130. <https://doi.org/10.1109/SP.2011.36>
- [12] Dongseok Jang, Zachary Tatlock, and Sorin Lerner. 2012. Establishing Browser Security Guarantees through Formal Shim Verification. In *USENIX Security Symposium*, Tadayoshi Kohno (Ed.). USENIX Association, 113–128.
- [13] Simon Holm Jensen, Magnus Madsen, and Anders Møller. 2011. Modeling the HTML DOM and Browser API in Static Analysis of JavaScript Web Applications. In *ACM SIGSOFT Symposium and European Conference on Foundations of Software Engineering (ESEC/FSE)*. ACM, New York, NY, USA, 59–69. <https://doi.org/10.1145/2025113.2025125>
- [14] Jeffrey J. Joyce and Carl-Johan H. Seger (Eds.). 1994. *Higher Order Logic Theorem Proving and Its Applications (HUG)*. LNCS, Vol. 780. Springer-Verlag, Heidelberg. <https://doi.org/10.1007/3-540-57826-9>
- [15] Gerwin Klein. 2009. Operating System Verification – An Overview. *Sādhanā* 34, 1 (Feb. 2009), 27–69.
- [16] Tobias Nipkow, Lawrence C. Paulson, and Markus Wenzel. 2002. *Isabelle/HOL—A Proof Assistant for Higher-Order Logic*. LNCS, Vol. 2283. Springer-Verlag, Heidelberg. <https://doi.org/10.1007/3-540-45949-9>
- [17] Azalea Raad, José Fragoso Santos, and Philippa Gardner. 2016. DOM: Specification and Client Reasoning. In *Programming Languages and Systems (LNCS)*, Atsushi Igarashi (Ed.), Vol. 10017. 401–422. [https://doi.org/10.1007/978-3-319-47958-3\\_21](https://doi.org/10.1007/978-3-319-47958-3_21)
- [18] Gareth David Smith. 2011. *Local Reasoning about Web Programs*. Ph.D. Dissertation. Imperial College London, London, UK.
- [19] Christian Sternagel and Renè Thiemann. 2014. XML. *Archive of Formal Proofs* (Oct. 2014). <http://isa-afp.org/entries/XML.shtml>, Formal proof development.
- [20] Peter Thiemann. 2005. A Type Safe DOM API. In *Database Programming Languages (LNCS)*, Gavin M. Bierman and Christoph Koch (Eds.), Vol. 3774. Springer, 169–183. [https://doi.org/10.1007/11601524\\_11](https://doi.org/10.1007/11601524_11)
- [21] W3C. 2015. W3C DOM4. (19 Nov. 2015). <https://www.w3.org/TR/dom/>
- [22] Markus Wenzel and Lawrence C. Paulson. 2006. Isabelle/Isar. In *The Seventeen Provers of the World, Foreword by Dana S. Scott (LNCS)*, Freek Wiedijk (Ed.), Vol. 3600. Springer, 41–49. [https://doi.org/10.1007/11542384\\_8](https://doi.org/10.1007/11542384_8)
- [23] WHATWG. 2017. DOM – Living Standard. (24 March 2017). <https://dom.spec.whatwg.org/commit-snapshots/6253e53af2fbfaad25ad09fd54280d8083b2a97/> Last Updated 24 March 2017.